

Strategic Delegation and Complement Public Goods*

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The paper presents a simple model of public goods provision in two regions and discerns between regimes of centralization and decentralization of political decision making. Public goods have specific feature because of being complement goods. The production of public good in one region induces positive externality in the other region. We show that in centralization there will be huge overprovision of public goods in comparison with social optimum, however in decentralization none of local governments provides any public good.

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1 Introduction

Centralization of political decision making is very large topic and there is a lot of empirical studies and theoretical models concerning advantages and disadvantages of the centralization. Oates (1972) in his decentralization theorem illustrates why the centralization can

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lead to suboptimal policies. It simply states that "...in the absence of cost-savings from the centralized provision of a [local public] good and of interjurisdictional externalities, the level of welfare will always be at least as high (and typically higher) if Pareto-efficient levels of consumption are provided in each jurisdiction than if *any* single, uniform level of consumption is maintained across all jurisdictions" (Oates 1972, p. 54). The costs of centralization stem from the policy uniformity when the diversity of preferences of agents in regions is neglected, whereas the benefits consist in economies of scale and internalization of externalities.

Ellingsen (1998) models the trade-off between the benefits and the costs of the centralization and determines the equilibrium design of jurisdictions. He illustrates that the relative size of the regions and the distribution of preferences are the key determinants of equilibrium. The work of Alesina and Spolaore (1997) is closely related, because they consider the trade-off between economies of scale and regional heterogeneity. In addition, they explore the influence of the democratization on the size of the government. Above mentioned studies in the Oates' tradition explain the cost of centralization in policy domains where public goods can not be differentiated according to the preferences of jurisdictions. However, in many cases it is possible to decide centrally on differentiated levels of public goods in regions according to the diverse preferences.

Centralized provision of local public goods when regions can be provided with different amounts is studied in Persson and Tabellini (1994). They state that it creates a free-rider problem which enhances the incentives of each region to lobby for federal spending. All agents in all localities have strong incentives to push for higher amount of public good, since they pay only a fraction of the costs. Nash equilibrium thus involves overprovision of all local public goods.

Unique approach is examined in Redoano and Scharf (2004). They compare policy centralization outcomes of public goods provision under alternative democratic choice procedures direct democracy and representative democracy and conclude that centralization is more likely to occur if the choice to centralize is made by elected policymakers than by referendum. In this situation, centralized policy is close to the preferred level of the region that least desires centralization.

Lockwood (2005) explores further arguments in favour of decentralization like higher

preference-matching and accountability of government. He surveys contributions to the study of fiscal decentralization using political economy approach and shows formal models which provide insights into when decentralization may fail to deliver these benefits.

However, none of the above-mentioned studies takes into account the possible effect of strategic delegation. This concept describes a situation when a voter with particular preferences elects a politician with different preferences from her own. It is a special case of strategic voting which occurs in more-rounds and more-proposals elections. Besley and Coate (1997) begin to analyze the strategic delegation in the case of cooperative decisions. They develop an alternative model of representative democracy and conclude that all decisions by voters, candidates and policy-makers are derived from optimizing behaviour. Voters may have therefore incentive to elect candidate with different preferences from their own if it coincides with their optimizing behaviour. In a later paper, Besley and Coate (2003) illustrate the trade-off between centralized and decentralized provision of local public goods in the case of spillover effects. They emphasize the importance of the decision-making mechanism in centralization, because voters may delegate policy making authority strategically. If the costs are shared through a common budget, voters have an incentive to delegate bargaining to public good lovers. Since in equilibrium all regions send public good lovers, the policy outcome will not be effective and the overprovision of public goods may occur. Dur and Roelfsema (2005) extend this analysis by allowing for non-shareable costs in the centralized public goods provision and show that under certain conditions voters delegate conservatives instead of public good lovers. Consequently, there will be the underprovision of public goods.

Jennings and Roelfsema (2004) apply this analysis to conspicuous public goods. Production of a conspicuous public good in one region has negative externalities in another region. In decentralized system, median voter elects a politician with lower preferences for conspicuous public good. On the contrary, in the centralization the delegated politician will have higher preferences. Roelfsema (2004) specifically considers the strategic delegation in the case of the environmental policy making. He argues that in a non-cooperative policy making setting voters may have an incentive to delegate politicians who care more for the environment than they do themselves. If voters anticipate cooperative policy making, they have an incentive to elect persons who care less for the environment. Another

application of strategic delegation provides Brueckner (2001) investigating the political economy effects of two different regimes of international capital taxation – tax competition and tax coordination. In the competitive tax regime, delegates simultaneously choose tax rates on capital maximizing utility of their outcomes independently and in the case of tax coordination they choose tax rates to maximize the sum of utilities. The policy makers then like the public good more than the median voters under competitive regime, therefore delegation has a tax-increasing effect. On the contrary, in coordination regime, voters delegate conservatives and taxes are lower.

The papers above use the cooperative bargaining and consider simple decision-making in centralized body. Segendorff (1998) computes another model of strategic delegation showing how the choice of particular type of agent can create a threat to the other nations agent. To find a bargaining outcome, instead of maximum of the sum of utilities, he implements the theory of Nash bargaining solution using reservation utilities. The author distinguishes between two cases, weak and strong delegation games. Weak delegation means that the delegated agents have no influence on the breakdown allocation and principals ideal allocations are implemented. Strong delegation gives each agent the authority to decide on the national breakdown allocation. He concludes that in the strong delegation game, delegated agent with less taste for the public good will decrease the reservation level of utility of the other nations agent and therefore the principal threatens the other countrys agent by her choice. In case of weak delegation, both principals are better off than in decentralized system. In the strong delegation game both principals delegate strategically and as a consequence the agreed allocation may provide less public good than under decentralized system.

The model in our paper is complementing the paper by Dur and Roelfsema (2005). Under similar assumptions, we consider public goods to be complements and we therefore show the strategic delegation effect for this type of goods. First section gives assumptions and then we derive the optimal solutions for decentralized decision making, social optimum and centralized decision making. However, in case of centralization we will find that the given assumptions are not sufficient to derive solution, consequently we specify them more.

2 Assumptions

Building on the framework of Dur and Roelfsema (2004) we construct the base model. The model concerns political decision making on public goods provision in two regions and describes voters incentives to delegate strategically.

Let us assume that regions are identical and denoted $i, i = 1, 2$. Individuals in each region differ in their preference λ for public goods, symmetrically distributed over the interval $\langle \underline{\lambda}, \bar{\lambda} \rangle$, in both regions identically. Symmetric distribution stands for the similarity of an individual with median preference and an individual with average preference, i.e. $\lambda^m = \frac{\underline{\lambda} + \bar{\lambda}}{2}$, λ^m denoting median preference.¹ The higher is an individual's λ , the stronger is her preference for public goods.

The region i produces a local public good g_i which entails utility for its citizens, however, its provision has also positive spillover effect on the utility of individuals in the region $-i$. The presence of the spillover effect is indicated by parameter $\kappa, \kappa \in \langle 0, 1 \rangle$. If $\kappa = 0$, spillover effect does not exist and individuals in region i do not get any utility from the provision of public good in region $-i$. The larger is the κ , the more the provision of g_i increases utility of individuals in $-i$. If $\kappa = 1$, individuals care equally for the public good g_i provided in their region as for the public good g_{-i} produced in the other region. This situation can be considered as the special case of global public goods.

The production of the public good is financed through non-distortionary income taxes. For simplicity we assume that the production of the public good has constant returns to scale, namely constant marginal costs, therefore tax costs are linear in the produced amount of public goods. To provide one unit of the public good, it is necessary to collect tax p from each individual in the region. Additionally, each unit of public goods produced in a region entails indirect utility cost c for each citizen in the region, and we suppose that also these costs are linear in public goods production. Since the regions are identical, $p_i = p_{-i} = p$ and $c_i = c_{-i} = c$.

¹Let λ^j denotes an individual's preference in the given region and n is the number of individuals in the region, then $j = 1, 2, \dots, n - 1, n$. Symmetrical distribution of preferences over the interval $\langle \underline{\lambda}, \bar{\lambda} \rangle$ indicates that the set of individuals preferences $N^\lambda, \lambda^j \in N^\lambda$ is $N^\lambda = \lambda^1, \lambda^2, \dots, \lambda^{n-1}, \lambda^n = \lambda^m + \alpha_1, \lambda^m + \alpha_2, \dots, \lambda^m + \alpha_{\frac{n-1}{2}}, \lambda^m, \lambda^m - \alpha_{\frac{n-1}{2}}, \dots, \lambda^m - \alpha_2, \lambda^m - \alpha_1$, where $\alpha_k \in \langle 0, \bar{\lambda} - \lambda^m \rangle, k = 1, 2, \dots, \frac{n-1}{2}$.

Average preference λ^a can be computed as $\lambda^a = \frac{\sum_{j=1}^n \lambda^j}{n} = \frac{n\lambda^m}{n} = \lambda^m \Rightarrow \lambda^m = \lambda^a = \frac{\underline{\lambda} + \bar{\lambda}}{2}$.

There is a major difference between the tax cost p and the indirect cost c . In centralized system, the tax cost can be shared among regions through a common central budget, but indirect cost cannot. This occurs when the cost c is closely related to the particular region and compensations are not feasible. How to interpret such type of the cost? It can be explained as some kind of negative externality associated only with the region where the production of public good is realized. We can imagine it as a decrease of a utility because some natural resource is damaged while producing the public good. As an example we can consider cutting down the trees to clear the area for building motor highway which causes harm to citizens like a loss of the lovely nature or reduction of oxygen which is not usually compensated by any transfer from the common centralized budget. Another way how to explain the indirect cost is in relation to health. The production of the public good can generate unhealthy conditions. Although we benefit from motor highway for number of years, as a consequence of air pollution, our health can get worse.

We already know the cost side in the utility function for region i which amounts to $t_i + cg_i$, where $t_i = pg_i$ in decentralization and $t_i = \frac{p}{2}(g_i + g_{-i})$ in centralization because tax costs are shared ², but we have not considered yet how the individuals value the public goods, i.e. the utility function.

Dur and Roelfsema (2004) use the additively separable utility function which means that individuals value separately public good provided in their region and public good produced in the other region. However, we examine an alternative specification of the utility function where local public goods are complements. It is a very special case, which is difficult to interpret, but it leads to interesting results. As an example, we can consider border protection. In such system like the Schengen is, the individuals utility of the border protection in each region depends on the minimal level of protection all regions set. The utility function is given as:

$$U_i^j = \lambda_i^j b\left(\min\{g_i, \kappa g_{-i}\}\right) + y - t_i - cg_i$$

Utility maximizing level of g_i depends on g_{-i} as in the previous case. However now the optimum satisfies condition $g_i \leq \kappa g_{-i}$ and policy maker will set g_i^* according to the

²Recall we assume that regions are identical, therefore $p_i = p_{-i} = p$ and $c_i = c_{-i} = c$.

first-order condition for $g_i = \min\{g_i, \kappa g_{-i}\}$. For $\kappa = 1$ we have $g_i \leq g_{-i}$ and for $\kappa \in (0, 1)$ g_i will be always strictly smaller than g_{-i} . If there is no spillover effect, then $g_i = g_{-i} = 0$. The utility maximizing level of g_i increases in g_{-i} up to g_i^* then it stabilizes at the level g_i^* and for given $g_{-i} \leq \frac{1}{\kappa}g_i^*$ the utility is first increasing up to $g_i = \kappa g_{-i}$ and then decreasing in g_i . Figure 1 shows indifference curves for this case.

We look for interior solutions, therefore we assume that gross income y is always sufficiently high to cover the total tax cost which the provision of public goods entails whatever amount the policy makers will decide on.

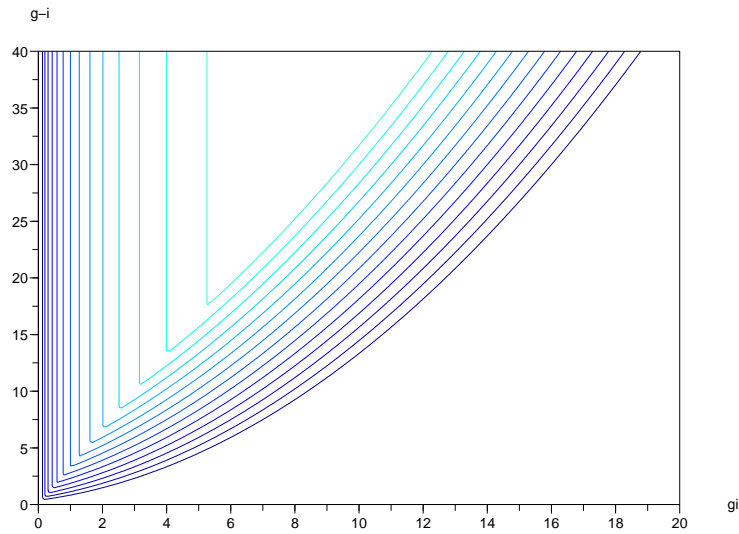


Figure 1: Complements ($\kappa = 0.3, \lambda = 0.6, p + c = 0.1, b(\cdot) = (\cdot)^{\frac{1}{2}}$)

3 Social optimum

In this section, we will determine the socially optimal amounts of the local public goods in both regions. We apply the utilitarian measure that the social optimum is defined as the outcome which maximizes the sum of utilities of all individuals in both regions. For computing the social optimal levels, we use the fact that under condition of symmetrical distribution of preferences it is possible to find g_i^*, g_{-i}^* maximizing sum of utilities of citizens in both regions, because we have $\{g_i^*, g_{-i}^*\} = \arg \max (U_i^m + U_{-i}^m)$, where U_i^m denotes the utility of the individual with median preferences in region i .

If individuals in region $i, i = 1, 2$, are symmetrically distributed over interval $(\underline{\lambda}, \bar{\lambda})$, in both regions identically, we have: $V_i = \int_{\underline{\lambda}}^{\bar{\lambda}} [\lambda_i^j b(g_i, g_{-i}, \kappa) + y - (p + c)g_i] d\lambda_i^j$

$$\begin{aligned}
V &= V_i + V_{-i} \\
&= \int_{\underline{\lambda}}^{\bar{\lambda}} [\lambda_i^j b(g_i, g_{-i}, \kappa) + y - (p + c)g_i] d\lambda_i^j + \int_{\underline{\lambda}}^{\bar{\lambda}} [\lambda_i^j b(g_{-i}, g_i, \kappa) + y - (p + c)g_{-i}] d\lambda_i^j \\
&= \int_{\underline{\lambda}}^{\bar{\lambda}} [\lambda_i^j b(g_i, g_{-i}, \kappa) + y - (p + c)g_i + \lambda_i^j b(g_{-i}, g_i, \kappa) + y - (p + c)g_{-i}] d\lambda_i^j \\
&= \int_{\underline{\lambda}}^{\bar{\lambda}} [\lambda_i^j (b(g_i, g_{-i}, \kappa) + b(g_{-i}, g_i, \kappa)) + 2y - (p + c)(g_i + g_{-i})] d\lambda_i^j \\
&= \left[\frac{(\lambda_i^j)^2}{2} (b(g_i, g_{-i}, \kappa) + b(g_{-i}, g_i, \kappa)) + (2y - (p + c)(g_i + g_{-i})) \lambda_i^j \right]_{\underline{\lambda}}^{\bar{\lambda}} \\
&= (\bar{\lambda}^2 - \underline{\lambda}^2) \frac{1}{2} (b(g_i, g_{-i}, \kappa) + b(g_{-i}, g_i, \kappa)) + (\bar{\lambda} - \underline{\lambda}) (2y - (p + c)(g_i + g_{-i})) \\
&= (\bar{\lambda} - \underline{\lambda}) \left[(\bar{\lambda} + \underline{\lambda}) \frac{1}{2} (b(g_i, g_{-i}, \kappa) + b(g_{-i}, g_i, \kappa)) + (2y - (p + c)(g_i + g_{-i})) \right]
\end{aligned}$$

Using $\lambda^m = \frac{\bar{\lambda} + \underline{\lambda}}{2}$ we get:

$$\begin{aligned}
V &= (\bar{\lambda} - \underline{\lambda}) \left[\lambda^m (b(g_i, g_{-i}, \kappa) + b(g_{-i}, g_i, \kappa)) + (2y - (p + c)(g_i + g_{-i})) \right] = (\bar{\lambda} - \underline{\lambda}) (U_i^m + U_{-i}^m) \\
&\Rightarrow \{g_i^*, g_{-i}^*\} = \arg \max V = \arg \max [U_i^m + U_{-i}^m]
\end{aligned}$$

To obtain the social optimum for complements, we have to maximize function:

$$\lambda^m \left(b(\min\{g_i, \kappa g_{-i}\}) + b(\min\{g_{-i}, \kappa g_i\}) \right) + 2y - (p + c)(g_i + g_{-i})$$

This function is maximized if and only if $g_i = g_{-i} = g$. The optimal level of public good satisfies the first-order condition:

$$\lambda^m \kappa b'(\kappa g) - (p + c) = 0$$

For $\kappa > 0$ we have $g^{SO} > g^D$, so if there is any positive externality of production of public goods, it is efficient to centralize production.

4 Decentralization of decision making

Under decentralized decision making, each region decides independently on its provision of local public goods. The production of public goods is financed through the local taxes, thus $t_i = pg_i$. In the first stage, voters in the region elect their policy-maker, who will decide on the level of the public good in the second stage. The elected policy-maker sets the level of g_i such that she maximizes her utility function. There is no additional incentive for re-election or a career promotion for policy-maker. The amount of production of local public good thus depends on the policy maker's preference λ .

When local public goods are complements, the objective function of the elected policy maker is $U_i^d = \lambda_i^d b(\min\{g_i, \kappa g_{-i}\}) + y - (p + c)g_i$. The optimal amount of g_i will always comply with $g_i \leq \kappa g_{-i}$. Let g_i^* denote the amount of public good satisfying $\lambda_i^d b'(g_i^*) - (p + c) = 0$. If $\kappa g_{-i} < g_i^*$, the utility maximizing level of g_i will be set as $g_i = \kappa g_{-i}$. If $\kappa g_{-i} \geq g_i^*$, the policy maker will maximize her utility function by providing g_i^* . In this situation we get into the same case as with additive separable utility function and the level of g_i^* is independent on g_{-i} . If we followed the condition $g_i = \kappa g_{-i}$ also in this situation, we would provide too much local public good.

Anticipating that the delegate will either choose g_i^* according to her preference or set $g_i = \kappa g_{-i}$, voters will not have any incentive to behave strategically, so they will elect the policy maker with median preferences.

Independent decision making in two regions about production of local public goods can be illustrated as a non-cooperative game, in which the policy-maker adjusts the amount of public good produced in her region according to the level in other region. The reaction curve of the policy-maker in region i is represented by the given first-order condition and it is best response function $BR(g_{-i})$. The Nash equilibrium of the game lies in the intersection of reaction curves and as Figure 2 shows, it satisfies $g_i^{NE} = g_{-i}^{NE} = 0$.

When local public goods are complements, neither region provides the local public good under decentralization. In comparison with two previous cases of utility functions, the voters will be now the worst off. This finding indicates that the centralization is desirable, especially when public goods are complements.

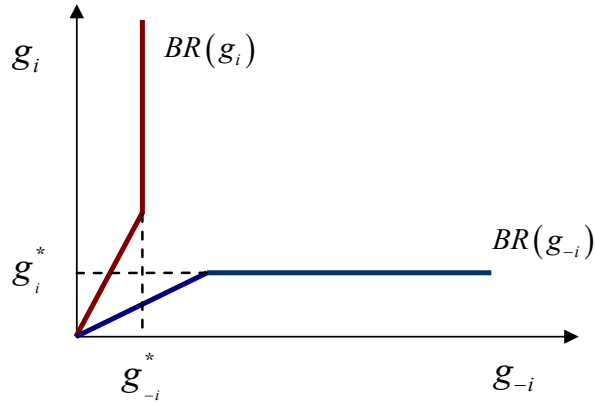


Figure 2: Reaction curves of decentralized decision making on provision of complements.

5 Centralization of decision making

We have demonstrated the difference between the amounts of public goods provided in decentralized systems and the social optimal amounts. The welfare maximizing levels of public goods are at least as high as those produced in the decentralization; for positive externalities, we even observe underprovision in decentralized systems. The remedy can be done by installing central body which will decide on local public goods provision in both regions. Central decision making has two stages. In the first stage, the voters in each region independently and simultaneously elect policy-makers from the regions' populations with preference $\lambda_i^d \in \langle \underline{\lambda}, \bar{\lambda} \rangle$; in the second stage, the elected policy-makers bargain over the amounts of public goods. We assume that bargaining is cooperative and the delegates maximize the sum of their utilities. The central government controls the common budget, through which the production of public goods is financed. Every individual in each region pays the tax cost $t_i = \frac{p}{2}(g_i + g_{-i})$.

If the delegation will be sincere and voters will not have any incentive for strategic voting, they would elect the agent with median preferences. In such case we would get into the social optimal situation and centralization would be pareto efficient. If there will be positive spillover effect, the centralization would be always welfare improving under assumptions of our model. We have to recall that this argument holds only for identical regions and we do not consider the trade-off between heterogeneity of preferences and internalization of externalities. Contrary to the analysis in Oates (1972), our model disregards the cost

side of centralization.

However, voters will not delegate an agent sincerely, because they have an incentive to misrepresent their policy preferences. To illustrate this, we use a non-cooperative example.

For complements, the objective function of two policy makers elected in both regions who bargain over the provision of public goods is:

$$U^d = \lambda_1^d b(\min\{g_1, \kappa g_2\}) + \lambda_2^d b(\min\{g_2, \kappa g_1\}) + 2y - (p + c)(g_1 + g_2) \quad (1)$$

We will solve the game by backward induction. In the second stage, we start by the fact for any fixed $g = g_1 + g_2$, there must be a unique $g_1^*(g)$. This allows us to split bargaining (in fact optimization of the joint utility function in (1)) into two steps: (i) recognizing function $g_1^*(g)$ and (ii) finding optimal g subject to $g_1^*(g)$. We find three candidate solutions.

In the first stage, we let voters elect the delegates. As usually, we use that $\lambda^d(\lambda_j)$ is monotonic in λ_j , so the median voter is decisive. Therefore, we can simplify the game into a non-cooperative game of two players, median voter in region 1 and median voter in region 2. In order to find a Nash equilibrium in pure strategies, we construct best responses of both players. We find an interesting equilibrium, and also provide constraints on function $b(\cdot)$ necessary for this equilibrium to sustain.

5.1 Delegates' optimum

We divide the optimization of (1) into two virtual steps. First, we let the delegates in the second stage jointly optimize on the constraint of a total amount fixed in the first period, namely $g_1 + g_2 = g$. Where is the optimum g_1 ?

By definition, $0 \leq g_1 \leq g$. The only problem is that complementarity violates monotonicity of the joint utility function. Therefore, we start by defining critical values in this interval in which the arguments within the minimum functions don't change, so the monotonicity is preserved. There are two critical values, therefore three intervals with three monotonic utility functions:

$$g_1^L = \frac{\kappa g}{1 + \kappa} \quad g_1^H = \frac{g}{1 + \kappa}$$

1. When $g_1 \leq g_1^L$, we have $U^d = \lambda_1^d b(g_1) + \lambda_2^d b(\kappa g_1) + 2y - (p + c)g$. Obviously, this is maximized for the highest available g_1 , i.e. $g_1 = g_1^L$.

2. When $g_1^L \leq g_1 \leq g_1^H$, we have $U^d = \lambda_1^d b(\kappa(g - g_1)) + \lambda_2^d b(\kappa g_1) + 2y - (p + c)g$. FOC gives us $\frac{\lambda_1^d}{\lambda_2^d} = \frac{b'(\kappa g_1)}{b'(\kappa g_2)}$. Because $b'(\cdot)$ is a monotonous strictly decreasing function ($b'' < 0$), we have $\lambda_1^d > \lambda_2^d \implies g_1 < g_2$. By analogy, $\lambda_1^d < \lambda_2^d \implies g_1 > g_2$. Public lover, as a result, gets relatively less than a conservative delegate.
3. When $g_1 \geq g_1^H$, we have $U^d = \lambda_1^d b(\kappa(g - g_1)) + \lambda_2^d b((g - g_1)) + 2y - (p + c)g$. Obviously, this is maximized for the lowest available g_1 , i.e. $g_1 = g_1^H$.

In total, written in general form, we observe that the optimum is located on the interval $g_i \in \langle g_i^L, g_i^H \rangle = \langle \frac{\kappa g}{1+\kappa}, \frac{g}{1+\kappa} \rangle$. In other words, we can use only the middle interval, since $g_1 \geq \kappa g_2$ and symmetrically $g_2 \geq \kappa g_1$.

By rewriting $\frac{\kappa g}{1+\kappa} \leq \frac{g}{2} \leq \frac{g}{1+\kappa}$, we also observe that the symmetric (equal) solution always lies in this interval, as long as $0 < \kappa \leq 1$.

With this knowledge, we proceed to the second step, namely optimization on this interval. We write Lagrangian, where (1) is maximized with the two inequality constraints, $g_1 - \kappa g_2 \geq 0$, $g_2 - \kappa g_1 \geq 0$, and respective multipliers μ_1, μ_2 .

$$\mathcal{L} = \lambda_1^d b(\kappa g_2) + \lambda_2^d b(\kappa g_1) + 2y - (p + c)(g_1 + g_2) + \mu_1(g_1 - \kappa g_2) + \mu_2(g_2 - \kappa g_1) \quad (2)$$

The Kuhn-Tucker conditions with complementary slackness yield:

$$\frac{\partial \mathcal{L}}{\partial g_1} = \kappa \lambda_2^d b'(\kappa g_1) - (p + c) + \mu_1 - \mu_2 \kappa \leq 0 \quad g_1 \geq 0 \quad \frac{\partial \mathcal{L}}{\partial g_1} g_1 = 0 \quad (3)$$

$$\frac{\partial \mathcal{L}}{\partial g_2} = \kappa \lambda_1^d b'(\kappa g_2) - (p + c) + \mu_2 - \mu_1 \kappa \leq 0 \quad g_2 \geq 0 \quad \frac{\partial \mathcal{L}}{\partial g_2} g_2 = 0 \quad (4)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_1} = g_1 - \kappa g_2 \geq 0 \quad \mu_1 \geq 0 \quad \frac{\partial \mathcal{L}}{\partial \mu_1} \mu_1 = 0 \quad (5)$$

$$\frac{\partial \mathcal{L}}{\partial \mu_2} = g_2 - \kappa g_1 \geq 0 \quad \mu_2 \geq 0 \quad \frac{\partial \mathcal{L}}{\partial \mu_2} \mu_2 = 0 \quad (6)$$

This gives us $2^4 = 16$ types of candidate solutions. However, we eliminate the inconsistent candidates: (i) $g_1 = 0, g_2 = 0$, (ii) $g_1 > 0, g_2 = 0$, (iii) $g_1 = 0, g_2 > 0$. Group (ii) is inconsistent with $g_2 \geq \kappa g_1$. Also group (iii) can be eliminated due to inconsistency with $g_1 \geq \kappa g_2$. We also eliminate the perverse group (i), where the optimum is non-provision. We are thus left only with group (iv), where $g_1 > 0$ and $g_2 > 0$, so conditions (3) and

(4) are satisfied with equality. We can immediately focus on the candidate solution with $\mu_1 > 0$ and $\mu_2 > 0$ (both constraints are active), which is feasible only for perfect spillover ($g_1 = \frac{g_2}{\kappa} = \kappa g_2$). As we are interested only in incomplete spillovers ($\kappa < 1$), we disregard this case. As a result we have three types of solutions, one interior and two corner solutions.

5.1.1 Interior solution

Interior solution implies inactive constraints, i.e. $\mu_1 = \mu_2 = 0$. From (3) and (4), we get:

$$b'(\kappa g_1) = \frac{p+c}{\kappa \lambda_2^d} \qquad b'(\kappa g_2) = \frac{p+c}{\kappa \lambda_1^d} \qquad (7)$$

We can interpret the result such that the preference of delegate 1 determines the good in region 2 and at the same time, region 1 bears only part of the costs, namely marginal costs are $\frac{p}{2}$. As a result, we will observe that individuals will have tendency to nominate public lovers and we end up in overprovision. This is however only up to some point, because a too extreme public lover will switch Interior solution into Lower corner solution, where marginal cost increases.

5.1.2 Upper corner solution (H)

Consider $\mu_1 = 0$ and $\mu_2 > 0$. From (5) and (6), we have $g_2 = \kappa g_1$. Putting into (3) and (4) and eliminating μ_2 , we get:

$$\kappa \lambda_1^d b'(\kappa^2 g_1) + \lambda_2^d b'(\kappa g_1) = \frac{(p+c)(\kappa+1)}{\kappa} \qquad (8)$$

5.1.3 Lower corner solution (L)

Consider finally $\mu_1 > 0$ and $\mu_2 = 0$. This is a symmetric problem to the previous case, only $g_1 = \kappa g_2$ and the solution writes:

$$\lambda_1^d b'(g_1) + \kappa \lambda_2^d b'(\kappa g_1) = \frac{(p+c)(\kappa+1)}{\kappa} \qquad (9)$$

5.2 Voters' optimum

Voters optimize under expectation of either of solutions. Since their preference for the delegate is monotonic in λ_j , it will again be upon the median voter in region 1 and median voter in region 2, which delegates are nominated. In this game of two players, we get equilibrium by deriving best responses, namely $\lambda_1^{d*}(\lambda_2^d)$ and $\lambda_2^{d*}(\lambda_1^d)$. However, we have to be cautious, since best responses may yield different types of solutions.

To solve this problem, we start with median voter in region 1. We divide his strategy set into three subsets corresponding to each type of solution. In each subset, we find a (conditional) best response, which is the best response when strategies are restricted only to be drawn from the subset. We denote them $\lambda_1^L(\lambda_2^d)$, $\lambda_1^I(\lambda_2^d)$, and $\lambda_1^H(\lambda_2^d)$. Finally, we compare payoffs for each conditional best responses, and select “the best of the best”, namely the genuine best response.

But how can we divide the space $\lambda_1 \times \lambda_2$ into strategy subsets relevant for each solution? Below, we will see that there exist two functions (boundaries) $\lambda_1^{bH}(\lambda_2)$ and $\lambda_1^{bL}(\lambda_2)$. If $\lambda_1 \leq \lambda_1^{bH}(\lambda_2)$, we have Upper corner solution. This is a case when Interior solution violates the upper bound, namely provides too much g_1 comparing to g_2 , so we have to have $g_1 = \frac{g_2}{\kappa}$. For $\lambda_1^{bH}(\lambda_2) < \lambda_1 < \lambda_1^{bL}(\lambda_2)$, we have Interior solution. And if $\lambda_1^{bL}(\lambda_2) \leq \lambda_1$, we are in Lower corner solution. Here, Interior solution would violated the lower bound, namely provided too little g_1 comparing to g_2 , so we have to have $g_1 = \kappa g_2$.

We know that Interior solution applies if and only if $g_1 \in \langle \kappa g_2, \frac{g_2}{\kappa} \rangle$. Can we make some inference about which $(\lambda_1^d, \lambda_2^d)$ lead to the solution with this property?

Using (7) and monotonicity of $b'(\cdot) > 0$ we get boundary functions:

$$\lambda_1^{bH}(\lambda_2^d) = \lambda_2^d \frac{b'(g_2)}{b'(\kappa g_2)} < \lambda_2^d < \lambda_2^d \frac{b'(\kappa^2 g_2)}{b'(\kappa g_2)} = \lambda_1^{bL}(\lambda_2^d) \quad (10)$$

5.2.1 Conditional best response for Interior solutions

Median voter in region 1 maximizes $U_1^m = \lambda_1^m b(\kappa g_2) + y - \frac{p}{2}(g_1 + g_2) - c g_1$:

$$\frac{\partial U_1^j}{\partial \lambda_1^d} = (\lambda_1^j b'(\kappa g_2) \kappa - \frac{p}{2}) \frac{d g_2}{d \lambda_1^d} = 0$$

We use implicit function theorem in (7) and derive that

$$\frac{dg_2}{d\lambda_1^d} = -\frac{p+c}{\kappa^2(\lambda_1^d)^2 b''(\kappa g_1)} > 0.$$

Therefore, we need $\lambda_1^m b'(\kappa g_2) \kappa - \frac{p}{2} = 0$, which in combination with the second term in (7) results into

$$\lambda_1^d = \frac{2(p+c)}{p} \lambda_1^m. \quad (11)$$

Now, consider implications of symmetry in preferences. Due to symmetry, we have $\lambda^m - \underline{\lambda} = \bar{\lambda} - \lambda^m$, where $\underline{\lambda} > 0$. This gives that $2\lambda^m = \bar{\lambda} + \underline{\lambda} > \bar{\lambda}$. As a result, $\lambda_1^d = \frac{2(p+c)}{p} \lambda_1^m > 2\lambda^m > \bar{\lambda}$. Therefore, conditional best response in Interior solution is constant and writes

$$\lambda_1^{I*}(\lambda_2^d) = \bar{\lambda}. \quad (12)$$

In other words, more than majority of voters tend to delegate the extreme public lover if they can count on the existence of the Interior solution.

5.2.2 Conditional best response for Upper corner solutions

Median voter in region 1 maximizes $U_1^m = \lambda_1^m b(g_1) + y - \frac{p}{2}(g_1 + g_2) - cg_1$ on the upper boundary $g_1 = \frac{g_2}{\kappa}$:

$$\frac{\partial U_1^j}{\partial \lambda_1^d} = \frac{dg_1}{d\lambda_1^d} \left[\lambda_1^m b'(\kappa^2 g_1) - \left(\frac{p}{2} + c \right) \right] - \frac{p}{2} \frac{dg_2}{d\lambda_1^d} = 0$$

By applying the implicit theorem on (8) and on the upper boundary $g_2 = \kappa g_1$, we have

$$\frac{dg_1}{d\lambda_1^d} = -\frac{b'(\kappa^2 g_1)}{\kappa^2 b''(\kappa^2 g_1) + \lambda_2^d b''(\kappa g_1)} > 0,$$

and

$$\frac{dg_2}{d\lambda_1^d} = \kappa \frac{dg_1}{d\lambda_1^d}.$$

As a result, we write the conditional best response implicitly by a system of two equations, where g_1 from the first equation is used to get function $\lambda_1^H(\lambda_2^d)$ in the second equation,

replication (8):

$$\lambda_1^m b'(g_1) - \frac{p}{2}(1 + \kappa) - c = 0 \quad (13)$$

$$\kappa \lambda_1^H(\lambda_2^d) b'(\kappa^2 g_1) + \lambda_2^d b'(\kappa g_1) = \frac{(p+c)(\kappa+1)}{\kappa} \quad (14)$$

Of course, the conditional best response in Upper corner solution is limited by $\underline{\lambda} \leq \lambda_1^H(\lambda_2^d)$ and $\lambda_1^H(\lambda_2^d) \leq \lambda_1^{bH}$, so we have to write:

$$\lambda_1^{H*}(\lambda_2^d) = \max\{\min[\lambda_1^H(\lambda_2^d), \lambda_1^{bH}(\lambda_2^d)], \underline{\lambda}\} \quad (15)$$

5.2.3 Conditional best response for Lower corner solutions

Median voter in region 1 again maximizes $U_1^j = \lambda_1^j b(g_1) + y - \frac{p}{2}(g_1 + g_2) - c g_1$, but now on the lower boundary $g_1 = \kappa g_2$:

$$\frac{\partial U_1^j}{\partial \lambda_1^d} = \frac{d g_1}{d \lambda_1^d} \left[\lambda_1^j b'(g_1) - \left(\frac{p}{2} + c \right) \right] - \frac{p}{2} \frac{d g_2}{d \lambda_1^d} = 0$$

Applying the implicit theorem on (9) and on the restriction $g_1 = \kappa g_2$, we get

$$\frac{d g_1}{d \lambda_1^d} = - \frac{b'(g_1)}{\lambda_1^d b''(g_1) + \lambda_2^d \kappa^2 b''(\kappa g_1)} > 0,$$

and

$$\frac{d g_2}{d \lambda_1^d} = \frac{1}{\kappa} \frac{d g_1}{d \lambda_1^d}.$$

All in all, we use this and (9) to get the implicit expression of the conditional best response:

$$\lambda_1^m b'(g_1) - \frac{p}{2} \left(1 + \frac{1}{\kappa} \right) - c = 0 \quad (16)$$

$$\lambda_1^L(\lambda_2^d) b'(g_1) + \kappa \lambda_2^d b'(\kappa g_1) = \frac{(p+c)(\kappa+1)}{\kappa} \quad (17)$$

Of course, the conditional best response for Lower corner solutions is limited by $\lambda_1^L \leq \bar{\lambda}$ and $\lambda_1^{bL} \leq \lambda_1^L$, so we finally write:

$$\lambda_1^{L*}(\lambda_2^d) = \min\{\max[\lambda_1^L(\lambda_2^d), \lambda_1^{bL}(\lambda_2^d)], \bar{\lambda}\} \quad (18)$$

Conditional best responses for median voter in Region 2 are symmetric for both upper and lower corner solutions.

5.3 Equilibrium with public lovers

Without explicit derivation of $\lambda_1^H(\lambda_2^d)$, $\lambda_1^L(\lambda_2^d)$, $\lambda_2^H(\lambda_1^d)$, and $\lambda_2^L(\lambda_1^d)$, it is extremely difficult to compare payoffs in all conditional best responses and thereby determine the true best response. Instead, we find sufficient conditions for certain intuitive equilibrium to exist.

In pure strategies, we know that the only symmetric solution is $\lambda_1^d = \lambda_2^d = \bar{\lambda}$. This is because symmetric solutions are always in the strategy subset corresponding to Interior solutions, as (10) shows.

For $(\bar{\lambda}, \bar{\lambda})$ to be a Nash equilibrium, we need to prove that $\lambda_1^*(\bar{\lambda}) = \bar{\lambda}$ (the other best response is symmetric). We employ a special strategy—instead of calculating best responses for all types of solutions, we find condition under which strategy subsets for Upper corner solution and Lower corner solution become unfeasible due to domain of $\bar{\lambda}$, i.e. $\lambda \in [\underline{\lambda}, \bar{\lambda}]$.

5.3.1 Eliminating Upper and Lower corner subsets

When $\lambda_2^d = \bar{\lambda}$, we know that Lower corner subset is out of feasible set of preferences, since by (10), we have $\lambda_1^{bL} > \bar{\lambda}$.

We will do exactly the same thing with Upper corner subset. In other words, we derive when the strategy subset corresponding to this solution materializes out of feasible set of preferences, namely $\lambda_1^{bH}(\bar{\lambda}) < \underline{\lambda}$. As explained above, this will be sufficient (but not necessary) condition for $(\lambda_1^d, \lambda_2^d) = (\bar{\lambda}, \bar{\lambda})$ to be the Nash equilibrium.

We seek critical condition under which $\lambda_1^{bH}(\bar{\lambda}) = \underline{\lambda}$. The boundary function λ_1^{bH} is defined for situation when the Interior solution in (7) gives allocation of g_1 which is just on the upper boundary of the interval $\langle \kappa g_2, \frac{g_2}{\kappa} \rangle$, namely $\kappa g_1 = \frac{g_2}{\kappa}$. We use that in Interior solution, a change in λ_1 does not affect g_1 , only g_2 , so we can define \bar{g}_1 for $\lambda_2^d = \underline{\lambda}$:

$$b'(\kappa\bar{g}_1) = \frac{p+c}{\kappa\bar{\lambda}} \quad (19)$$

We use (7) to derive that for $\lambda_2^d = \underline{\lambda}$

$$\lambda_1^{bH}(\bar{\lambda}) = \bar{\lambda} \frac{b'(\kappa\bar{g}_1)}{b'(\kappa^2\bar{g}_1)}. \quad (20)$$

By using implicit definition of \bar{g}_1 in (19), we find equivalence:

$$\lambda_1^{bH}(\bar{\lambda}) < \underline{\lambda} \iff \bar{\lambda} \frac{p+c}{\kappa\bar{\lambda}} \frac{1}{b'(\kappa^2\bar{g}_1)} < \underline{\lambda} \iff b'(\kappa^2\bar{g}_1) > \frac{p+c}{\kappa\underline{\lambda}} \quad (21)$$

That is the condition we have been seeking. We can use it together with relation of upper boundary in (20) to discuss what the condition intuitively requires:

$$\lambda_1^{bH}(\bar{\lambda}) < \underline{\lambda} \iff b'(\kappa^2\bar{g}_1) > \frac{p+c}{\kappa\underline{\lambda}} \iff \frac{b'(\kappa^2\bar{g}_1)}{b'(\kappa\bar{g}_1)} > \frac{\bar{\lambda}}{\underline{\lambda}} \quad (22)$$

In other words, the condition requires that (i) either the population is sufficiently homogenous ($\underline{\lambda}$ being close enough to $\bar{\lambda}$), or (ii) demand for public good $b(\cdot)$ sufficiently elastic. The latter requirement is based on the fact that marginal utility $b'(\cdot)$ is monotonic (decreasing) and positive. High elasticity implies sufficiently responsive marginal utility; in other words sufficiently low $b''(\cdot)$.

5.3.2 Interpretation

For the equilibrium with strong public-good loving delegation, we need to impose two additional conditions:

1. $b'(0) > \frac{p+c}{\underline{\lambda}\kappa}$
2. $b' \left[\kappa b'^{-1} \left(\frac{p+c}{\kappa\underline{\lambda}} \right) \right] > \frac{p+c}{\kappa\underline{\lambda}}$

If even conservatives demand non-negative amounts of public good, if demand for public good is rather elastic and population sufficiently homogeneous, we found that cooperative centralization in case of complements with spillovers leads to strategic delegation of extreme public good lovers.

6 Conclusion

The model in the paper illustrates the effect of strategic delegation in case of complement public goods. We found that under decentralization of policy decision making, non of the regions provides any public good. However, it holds for any spillover effects except 1, in sense of public goods being global public goods. In such a situation, citizens in both regions value different public goods equally, the decentralized production is positive and we get to the social optimum.

For positive spillover effects, the centralized provision of public goods is always welfare superior. By centralization we mean that voters in each region elect a policy-maker into the central government and policy-makers then decide about provision of public goods. We considered case of utilitarian bargaining which means that politicians maximize sum of their utilities. To derive the solution, we imposed another restriction on the utility function. If its second derivation is sufficiently high then we showed that under centralized decision making voters delegate policy-maker strategically and they always choose extreme public good lover. We get to the same conclusion when imposing further restriction on interval of possible preferences for public good to be sufficiently narrow. This situation leads to huge overprovision of public good but the voters are better off than under decentralization.

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